

## **HOW TO STUDY MATH**

### **OBJECTIVES (for this unit & for your general math practice)**

- Learn the basics of mathematics language.
- Learn problem-solving skills in structured, sequential and systematic processes.
- Develop skills from the start rather than just memorizing terms without comprehending them.
- Develop skills to follow step-by-step sequences in doing math with accuracy.
- Develop understanding of the meanings and reasons for each mathematical sequence.
- Leave no gaps in your understanding of the day to day topics & exercises. Knowledge in math is cumulative. New concepts are built on the previous concepts. Stay up to date in your assignments.
- Develop an understanding of all concepts.

### **GROUNDWORK FOR A MATH CLASS**

- Have a good foundation in basic math skills, e.g.; numbers, fractions, decimals, integers, percentages, and ratios.
- Try to understand the meanings of math symbols and formulas.
- Have the ability to analyze given information and relate it to mathematical terms. e.g.; sum, simplify, factor, solve, graph.

### **TIPS FOR STUDYING FOR STUDENTS WHO ARE WEAK IN MATH**

- Follow step-by-step sequence of operations in doing a math problem. Each step is very precise according to mathematical rules.
- Read the math text book thoroughly.
- Do the example problems before starting exercise problems.
- Learning math is a sequential process. You should attend classes regularly.
- The best time to study is right after the class.
- Ask questions in class if you don't understand.
- Read and adopt the Math Anxiety Bill of Rights.
- Follow the two-column method to learn a new math topic.
- Seek help as soon as you encounter a problem.
- Never memorize--rather try to analyze
- Repetition is the key to understanding math. You should do at least one problem of each topic every day.
- Use the distributive learning method to study. Do problems at intervals. Study new and old materials every day.
- Maintain structured notes. When you encounter a new topic, document every step of the process.
- Maintain a positive attitude. Develop your internal locus of control by taking responsibility for your success.
- Have faith in yourself.
- Never put aside math work for the next day. Avoid procrastination.
- Review old class notes and get an overview of the new material before the class.

- Do not omit any step in the sequence; rather each step should be done according to mathematical principles.

### **GENERAL RELAXATION AND TREATMENT OF ANXIETY**

- Sit up in your chair in a relaxed position, head straight and hands uncrossed in your lap.
- Close your eyes and take 20-30 seconds to relax. Let go of any tension in your face, neck, and shoulders.
- Inhale, breathing deeply into your abdomen. Your stomach will expand when you breathe deeply.
- When you have filled your lungs with air, pause; purse your lips as if you were about to whistle; then exhale evenly and forcefully through the small hole between lips.
- At the end of your exhalation, pause, and then push out the last bit of remaining air in three short, forceful puffs.
- Repeat this process 3-5 times and talk to yourself...

I feel good.

I feel quiet and my hands are warm.

My attention is drawn inward and I am at ease.

Deep in the center of myself I feel quiet and peaceful.

My attention is drawn inward and I am peaceful and serene.

My arms and my hands are heavy, quiet, and warm.

I feel good.

I am at peace.

My heartbeat is calm and regular.

I can solve all math problems.

- When finished, sit quietly for a minute, observing the rise and fall of your abdomen as you breathe normally.
- Once you are in a relaxed state, picture yourself approaching your next math assignment with confidence. Feel the feeling of being in control of your anxiety. See yourself doing well on your next math exam. See yourself sitting down, and beginning to work on the problems feeling calm, secure, and unafraid. Carry this feeling of confidence with you when you approach your next math problem.

### **MATH DEMYSTIFIED**

Do words like “college algebra” and “calculus” fill you with foreboding? Or does your brain just go number with boredom whenever you sit down to do math homework? Many people can think of about a zillion things they’d rather do than work a math problem. Even simple addition drives many of us to our calculators.

There are people, however, with positive dispositions toward mathematics. What do they have that makes math easier for them? First of all, they approach math the same way they approach anything else – as a challenge that can be met. They don’t make it harder than it really is. Many people like math because knowledge in math is cumulative. New concepts are built on previous concepts. The great thing about this is that basic mathematical concepts aren’t difficult at all. Two plus two does equal four, and everything else in math just grows out of simple stuff like this. The trick is to develop an understanding of all concepts, going step by step.

You don't have to be intimidated by words like "coefficient" and "exponential." These aren't mystical terms; they have a meaning that you can understand and that is usually pretty simple. Math is easier if you comprehend terms from the start rather than just memorize them. Then you can develop an understanding of the meanings and reasons for each mathematical sequence. (Yes, they do have meanings and reasons. That's the great thing about math!)

To get off on the right foot in your math class, first dispel any myths you may hold about math (some are listed below). Then establish a solid foundation by reviewing math basics – remember, these are your building blocks, so you can't just forget what you've already learned.

### **The Myths of Math**

- Men are better in math than women.
- Math requires logic, not intuition.
- Math is not creative.
- You must always know how you got the answer.
- There is a best way to do a math problem.
- It's always important to get the answer exactly right.
- It's bad to count on your fingers.
- Mathematicians do problems quickly in their heads.
- Math requires a good memory.
- Math is done by working intensely until the problem is solved.
- Some people have a "Math Mind" and some don't.
- There is a magic key to doing math.

*Borrowed from **Mind Over Math** by Kogelman and Warren.*

### **MATH ANXIETY BILL OF RIGHTS**

I have the right to:

- learn at my own pace and not feel stupid if I'm slower than others.
- ask whatever questions I have.
- need extra help.
- ask a teacher/tutor for help.
- say I don't understand.
- not understand.
- feel good about myself regardless of my math abilities.
- not base my self-worth on my math skills.
- view myself as capable of learning math.
- evaluate my math teachers and how they teach.
- relax.
- be treated as a competent adult.
- dislike math.
- define success in my own terms.

*S.L. Davis, University of Minn.*

### **Understanding Orders of Operation**

Higher level math problems are just a series of simple operations. The key to getting the right answer is performing the operations in the right order. You start from the most interior part of the problem (operations in parenthesis) and work you way out [to brackets]. At each level (from parenthesis to brackets)

- First calculate the exponents
- Then the multiplication or division
- Then the addition or subtraction

Order of Operation Rules:

<p>1. Simplify inside the ( ), { }, [ ] .</p>	<p>Example:</p> $[4 \cdot \{23 + 6(5+5)\} - 43] + 1 =$ $[4 \cdot \{23+6(10)\} - 43] + 1 =$ $[4 \cdot \{23 + 60\} - 43] + 1 =$ $[4 \cdot \{83\} - 43] + 1 =$ $[332 - 43] + 1 =$ $289 + 1 = 290$
<p>2. Exponents</p>	<p>a. Positive number raised to an even exponent.</p> $(\ )^4 = \quad (\text{Ex: } (-2)^4 = 16)$ <p>b. Negative number raised to an odd exponent.</p> $(\ )^3 = \quad (\text{Ex: } (-2)^3 = -8)$ <p>c. Negative number raised to an even exponent. Negative is not included in calculating the answer.</p> $-A^2 = -C \quad (\text{Ex: } -2^2 = -4)$ <p>d. Negative <i>expression</i> raised to an even exponent (an expression will appear in parenthesis). <i>Negative</i> is included in the calculation as it is inside the parenthesis.</p> $(-A)^2 = +C \quad (\text{Ex: } (-2)^2 = +4)$ <p>e. <math>(a^x b^y)^m = a^{xm} b^{ym}</math>  Ex. <math>(x^2 y^3)^2 = x^4 y^6</math></p> <p>f. <math>a^0 = 1</math></p> <p>g. <math>a^{-x} = 1 / a^x</math></p>

<p>3. Multiplication and division Solve Left to Right, whichever comes first.</p>	<p>1. Multiplying same signs (two positive or two negative) answer is always positive (+)</p> <p>2. Different signs; answer is always negative (-)</p> <p>3. <math>a^x * a^y = a^{x+y}</math></p> <p>4. <math>a(b + c) = ab + ac</math> Distributive property</p> <p>5. <math>-(x-y) = -x + y</math></p> <p>6. <math>(a + b)(c + d) = ac + ad + bc + bd</math></p> <p>7. <math>a^x / a^y = a^{x-y}</math></p> <p>8. <math>(a + b) / c = a/c + b/c</math></p>
<p>4. Addition and subtraction Solve Left to Right, Whichever comes first.</p>	<p>1. Same signs, add the numbers and keep the same sign.</p> <p>2. Different signs, subtract the small from the large number and put the sign of the larger number.</p> <p>3. Add and subtract the coefficients of LIKE terms. Ex: <math>3x + 4x = 7x</math></p>

### Two Column Method

When learning a new mathematical procedure, it's helpful to arrange your information (notes/samples problems) into two columns. The first column explains what to do. The second column is for examples.

#### Solving a linear equation:

$$3(X - 1) + 2X = 6X - 3(X + 1) + 6$$

Mathematical sequence	Example
Simplify the left hand side and right hand side of the equation. Use distributive property to remove the parenthesis.	$3X - 3 + 2X = 6X - 3X - 3 + 6$
Add and subtract coefficients of like terms.	$5X - 3 = 3X + 3$
Move the variable-terms to left hand side and move terms without the variable to the other side. <i>When terms are moved, the sign of the terms will change.</i>	$5X - 3X = 3 + 3$
Simplify the left and right hand sides	$2X = 6$
Divide both sides by the coefficient of the variable.	$2X/2 = 6/2$
Simply both sides and get the solution of the the equation.	$X = 3$
Check the answer. Substitute the value of the variable in both sides.	$3(3 - 1) + 2(3) = 6(3) - 3(3 + 1) + 6$
Simplify inside the parenthesis.	$3(2) + 6 = 18 - 3(4) + 6$
Remove the parenthesis by multiplying.	$6 + 6 = 18 - 12 + 6$
Simplify left and right hand side. Because the left and right hand sides are equal, the solution is correct.	$12 = 12$

## **ANALYZING AND SOLVING WORD PROBLEMS**

1. Read through the problem at a moderate speed to get an overview.
2. Read through the problem again, and ask yourself the following questions.
  - a. What is the problem asking for? (Usually this information will be at the last part of the problem.)
  - b. What is the problem telling me that is useful? (At this point cross out any irrelevant information.)
  - c. What is the problem trying to say? (Analyze it in your own words or talk to yourself about the facts.)
3. Read the problem again to verify your understanding of it and form relationships among all facts given.
4. Draw pictures or diagrams to give you a visual aid.
5. Make a table or box with the given information and leave a blank space for the information you are looking for.
6. Take the unknown information as a variable. Make sure you write down what the unknown represents.
7. Translate the English terms into an algebraic equation. (Check the equation by translating it back into English.)
8. Solve the equation.
9. Verify your solution with the given problem.

**Word Problem Example 1:**

A coffee manufacturer wants to market a new blend of coffee that will cost \$2.90 per pound by mixing two coffees that sell for \$2.75 and \$3.00 per pound, respectively. What amounts of each coffee should be blended to obtain the desired mixture? (Assume the total weight of the desired blend is 100 pounds.)

Solution:

Asking for the amounts of Coffee #1 and Coffee #2

Given data:

\$ 2.90 per pound = the unit “per pound” means it is the unit cost  
it is the unit cost of the mixture

\$ 2.75 per pound = the unit “per pound” means it is the unit cost  
it is the unit cost of Coffee #1

\$ 3.00 per pound = the unit “per pound” means it is the unit cost  
it is the unit cost of Coffee #2

100 pounds = the unit “pounds” means weight  
it is the total weight of the mixture

Assume:

The weight of Coffee #1 =  $X$   
Then the weight of Coffee #2 =  $100 - X$

Notes:

Total Cost = Unit Cost \* Weight  
Work the problem using cents, rather than dollars.

	Coffee #1	Coffee #2	Mixture
Unit Cost	275	300	290
Weight	$X$	$100 - X$	100
Total Cost	$275X$	$300(100 - X)$	$290(100)$

Set the Equation:

Total Cost of Coffee #1 + Total Cost of Coffee #2 = Total Cost of Mixture

$$\begin{aligned}
 275X + 300(100 - X) &= 290(100) \\
 275X + 30,000 - 300X &= 29,000 \\
 -25X + 30,000 &= 29,000 \\
 -25X &= 29,000 - 30,000 \\
 -25X &= -1000 \\
 X &= 40
 \end{aligned}$$

Analysis:

$X =$  amount of Coffee #1 = 40 pounds  
amount of Coffee #2 =  $100 - X = 100 - 40 = 60$  pounds

Check:

$$\begin{aligned}
 275(40) + 300(60) &= 29000 \\
 11000 + 18000 &= 29000 \\
 29000 &= 29000
 \end{aligned}$$

Checked

**Word Problem Example 2:**

A motorboat heads upstream on a river that has a current of 3 miles per hour. The trip upstream takes 4 hours, while the return trip takes 2.5 hours. What is the speed of the motorboat in still water? (Assume the motorboat maintains a constant speed relative to the water).

Solution:

Asking for the speed of the motorboat in still water

Given data:

3 miles per hour = the unit "miles per hour" means rate  
 it is the speed of the current  
 4 hours = the unit "hours" means time  
 it is the time for the upstream trip  
 2.5 hours = the unit "hours" means time  
 it is the time for the return trip (down stream)

Assume:

The speed of the motorboat in still water = X

Notes:

Upstream speed of the boat  
 = Speed of the boat in still water - Speed of the current  
 Downstream speed of the boat  
 = Speed of the boat in still water + Speed of the current  
 Distance = Rate \* Time

	Rate	Time	Distance
Upstream	$X - 3$	4	$4(X - 3)$
Downstream	$X + 3$	2.5	$2.5(X + 3)$

Set the Equation:

Distances for upstream and downstream are the same

$$\begin{aligned}
 4(X - 3) &= 2.5(X + 3) \\
 4X - 12 &= 2.5X + 7.5 \\
 4X - 2.5X &= 7.5 + 12 \\
 1.5X &= 19.5 \\
 X &= 13
 \end{aligned}$$

Analysis:

X = The speed of the motorboat in still water = 13 miles per hour

Check:

$$\begin{aligned}
 4(13 - 3) &= 2.5(13 + 3) \\
 4(10) &= 2.5(16) \\
 40 &= 40 \quad \text{Checked}
 \end{aligned}$$

**English Words to Algebraic Expressions (be able to translate these signal words)**

+	-	X	/	=	>
Sum	Decreased by	Times	Per	Is	Greater than
Add	Less than	Percent of	Quotient	Was	> = Greater than or equal to
In addition	Subtract	Product	Divide	Will be	Results
More than	Difference	Interest on			< Less than
Increased	Diminished				< = Less than or equal to
In Excess	Reduce				
Greater	Remainder				

Two more than N	$N + 2$
A number added to 10	$10 + N$
A number increased by 15	$N + 15$
The sum of N and 5	$(N + 5)$
20 less than a number	$N - 20$
A number decreased by 64	$N - 64$
The difference of N and 5	$(N - 5)$
Difference between N and 6	$N - 6$
Difference between 6 and N	$6 - N$
Subtract 3 from a number	$N - 3$
Twice a number	$2N$
Ten percent of a number	$.10N$
Five times a number	$5N$
Quotient of N and 3	$(N/3)$

Quotient of 3 and N	$(3/N)$
Two times the sum of a number and 6	$2(N+6)$
Consecutive integers (N being the first integer)	N $N + 1$ $N + 2$
consecutive ODD Integers (N being the first odd integer)	N $N + 2$ $N + 4$
Consecutive EVEN Integers (N being the first even integer)	N $N + 2$ $N + 4$

**Formulas (Special Products)**

$(A + B)^2 = A^2 + 2AB + B^2$	$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$
$(A - B)^2 = A^2 - 2AB + B^2$	$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$
$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$	$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
$A^2 - B^2 = (A + B)(A - B)$	$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

## **TEST TAKING SKILLS**

### Components of Test Anxiety:

- Cognitive test anxiety  
The students' concerns about their academic performance, e.g.; failing the test, not maintaining an A, not smart enough for this class, never been good at math.
- Physiological test anxiety  
The attention of the students to their increased physiological arousal, e.g. sweaty palms, tense muscles, rapid breathing, nervousness, double vision, headache, stomach upset.

### **Before the Test**

1. Organize all your homework and class notes that will cover the materials for the test.
2. Start studying at least one week ahead of the scheduled test day.
3. Hold group study sessions with classmates or extra sessions with a tutor.
4. Develop practice tests or ask the instructor for tests from previous semesters.
5. Note similarities and differences among problems.
6. Locate additional problems and use them to take a practice test.
7. Don't study right before the test.
8. Get plenty of rest before you take the test.
9. Develop reasonable learning and study strategies.

### **During the Test**

1. Glance over the whole exam quickly, assessing questions as to their level of difficulty and point value.
2. Do a data dump on the test – which means write down any information, formulas, definitions that will be helpful for you to answer any question.
3. Do some relaxation techniques while you are taking the test.
4. Answer the problems which you can solve, and make sure you are correct.
5. Attempt the harder problems during the latter part of the test.
6. Maximize partial credit by showing all your work.
7. Review the answers before you turn in your test.

### **Common Pitfalls During Tests**

- careless errors
- waste of time
- lack of understanding
- uncertainty of which method to use
- lack of pre-requisite information
- test anxiety

### **After the Test**

1. Change of study habits to improve test performance.
2. Analyze test questions for future tests.

### **Tips to Reduce Stress/Anxiety**

- Deep breathing
- Foot massage
- Positive self-statements or self-talk:
  - “I can do this”
  - “I have studied, and I know I can do this math problem”
- Visualization;
  - Picture yourself feeling calm and confident.
- Keep a positive attitude and relax.
- Concentrate on the test; don’t worry about your ability or what you did not study.

Good luck! Don’t hesitate to get in touch with an APSS counselor if math continues to be an unbearable experience for you. With continued effort—and by committing yourself to the strategies reviewed here (including the use of math tutors and study buddies)—you will discover that math holds as much for you as it does for the other “math wizards” in your life!